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On fermion mass hierarchy with extra dimensions

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Abstract

Recently, various phenomenological implications of the existence of extra space-time dimensions have been investigated. In this letter, we construct a model with realistic fermion mass hierarchy with (large) extra dimensions beyond the usual four dimensions. In this model, it is assumed that some matter fields live in the bulk and the others are confined on our four dimensional wall. It can naturally reproduce the quark and lepton mass hierarchy and mixing angles without any symmetry arguments. We also discuss some possibilities of obtaining suitable neutrino masses and mixings for the solar and atmospheric neutrino problems.

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To understand the hierarchical structure of the fermion masses and mixings is one of the most important problem in the particle physics. In the Standard Model, all the Yukawa couplings are completely free and they are nothing but phenomenological input parameters. To reproduce fermion masses or at least to relate one mass (Yukawa coupling) to others the informations on some dynamics at higher scale are needed beyond the Standard Model. One important approach is to consider grand unified theories (GUT), in which frameworks the quarks and leptons are also unified into the same multiplets of unified gauge groups and their Yukawa couplings are related each other at high energy. This is a very beautiful approach, but still there are large ambiguities in determining the Yukawa couplings of three generations. In addition, the GUT gauge symmetries generally relate quarks and leptons and so their mixing angles. This fact simply induces a contradictory small lepton mixing, and then we have to deal with rather complex models to reproduce realistic low-energy values of couplings. In $SU(5)$ GUTs, however, the left-handed (charged-) leptons are associated with only the right-handed down quarks, not with the left-handed quarks, resulting in possible large lepton mixing without having unrealistic CKM angles. This interesting feature has been recently discussed and used to construct models with large lepton mixing [1].

Recently, the notable approach to models beyond the Standard Model is proposed [2, 3, 4]. This approach invokes the existence of large extra dimensions in addition to our four dimensions. This interesting possibility has been used to argue many problems in the present particle physics and astrophysics [5], and its various effects on future experiments are also intensively discussed [6]. With these large extra dimensions, we can have many renewed insights into those have been discussed in traditional four dimensional frameworks; the gauge hierarchy problem [4], gauge coupling unification [3, 7], fermion mass hierarchy [3, 8, 9], neutrino physics [10], proton decay, and so on.

In this letter we demonstrate that in a GUT framework with (large) extra dimensions, the observed fermion mass hierarchy and mixing angles which include the large lepton mixing can be naturally reproduced. As for the size of the extra dimensions R, there has been many works in which R^{-1} is much lower than the four-dimensional Planck scale; $R \sim O(\mathrm{mm})$ or $O(\mathrm{TeV}^{-1})$. In such frameworks, many new interesting possibilities occur as mentioned above. On the other hand, for the most part of this paper we consider the compactification of the extra dimensions occurs around the usual GUT scale $\sim 10^{16}$ GeV and the fundamental scale M_* is taken as the order of 10^{18} GeV. Taking the compactification scale R^{-1} near the GUT scale also has several attractive features

from phenomenological points of view. Below the GUT scale, we have the Minimal Supersymmetric Standard Model (MSSM) and so the traditional gauge coupling unification by interpolating the experimental data [11] can be preserved. In addition, the Kaluza-Klein (KK) threshold corrections at the GUT scale may improve a little larger prediction of the low-energy $SU(3)$ gauge coupling in the MSSM [12]. The proton stability can be also guaranteed more trivially than in the low-scale unification scenarios, and the supersymmetry breaking problems (the flavor problem, etc.) and many other problems could be solved by considering the compactification just below the Planck scale [13].

To obtain hierarchy among Yukawa couplings, we need have the origins of suppression of these couplings. In the traditional four dimensional models, these small factors originate from some hierarchies among couplings themselves, a small ratio to the fundamental scale, the radiative corrections, and so on. It implies that we have to invoke some specific dynamics such as flavor symmetries in order to have hierarchical couplings. On the other hand, in models with extra spacetime dimensions, we generally have some possible suppression mechanisms for couplings. First possibility is a volume factor accompanied with each bulk field in integrating out the extra dimensions [5, 10]. In case of one extra dimension, this volume factor ϵ associated with its mode expansion becomes

$$\epsilon \simeq \frac{1}{(M_* R)^{1/2}}\,, \tag{1}$$

to have a canonically normalized four dimensional field. For the fields which can propagate in n extra dimensions, this volume factor becomes $1/(M_* R)^{n/2}$. Therefore, when there are different fields propagating in different numbers of dimensions, different volume factors are attached to their couplings in four dimensions. If the size of the extra dimensions is relatively large, this factor can produce hierarchy among couplings. Secondly, the other possible effect is that some couplings have the power-law evolution behaviors in their renormalization group equations (RGE) due to the existence of infinite KK towers in the bulk [14, 3, 8]. In other words, this is due to the fact that the higher dimensional couplings have non-zero classical mass dimensions and then under the classical scaling, each coupling suffers the power-law corrections. This large effect can vary the low-energy values of coupling constants considerably and in some cases we can obtain hierarchical structures among these couplings in the form of $1/(M_* R)^x$ as well as the first possibility. Though other interesting mechanisms which depend on some specific dynamics have been proposed so far [9], in this paper we focus on the first possibility. We will also mention that the second approach, namely, the RGE effects can be used alternatively to obtain the same results as in the models constructed by the first approach.

Let us explain our model. We consider a supersymmetric $SU(5)$ GUT model with 3 generations $\mathbf{10}$ and $\mathbf{5}^*$ representation matters and 3 singlets as the right-handed neutrinos. Among these matter fields, we assume that the first generation $\mathbf{10}_1$ can propagate in two extra dimensions (the fifth and sixth) and the second generation $\mathbf{10}_2$, one extra (the fifth) dimension. The $SU(5)$ gauge fields are also in the bulk and all other fields are confined on the four dimensional wall (our world). This configuration, in which the different fields feel the different numbers of extra dimensions, can be actually realized in the Type I string theory [3]. The extra dimensions are compactified on tori with a common radius R and its scale is supposed to be on the order of the GUT scale ($R^{-1} \simeq 10^{16}$ GeV). In this situation, we have suppression factors in the interaction terms concerning the $\mathbf{10}_{1,2}$ fields due to the above mentioned volume factor; they are ϵ^2 accompanied with $\mathbf{10}_1$ and ϵ^1 with $\mathbf{10}_2$. Since each Yukawa coupling is given by the superpotential $\mathbf{10}_i\mathbf{10}_jH_u$ for the up-type quarks, $\mathbf{10}_i\mathbf{5}^*_jH_d$ for the down-type quarks and the charged leptons, and $\mathbf{5}^*_i\mathbf{1}_jH_u$ for the neutrinos, the order of magnitude of the quark and lepton Yukawa couplings just below the GUT scale ($\simeq$ the compactification scale) become as follows;

$$Y_u \simeq \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \tag{2}$$

$$Y_d \simeq \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon & \epsilon & \epsilon \\ 1 & 1 & 1 \end{pmatrix}, \tag{3}$$

$$Y_e \simeq \begin{pmatrix} \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \tag{4}$$

$$Y_\nu \simeq \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{5}$$

These matrices are denoted in the basis of $L_iY_{x_{ij}}R_j$ form where $L(R)$ represents the left(right)-handed fermions. It should be noted that since the gauge fields are assumed to live in the bulk, the four-dimensional gauge coupling constant is only normalized as $g \to g/\sqrt{V}$ (V: the volume of bulk) independently of the properties of the matter fields. Therefore, we have no hierarchical strength of gauge coupling to different matter fields. In deriving the above matrix forms we assume that all the Yukawa couplings are of the same

order of magnitude in higher dimensional (higher scale) theories, which can be naturally expected. Of course, there also exist other possibilities of taking specific forms of Yukawa couplings which may be restricted by the group theoretical assumptions, some flavor symmetries, etc. Such choices could improve the inaccurate relations in detailed analyses but we do not mention this possibility in this letter.

In the Yukawa matrices (2)–(5), we have an ambiguity of $O(1)$ coefficient in each element. Therefore each matrix is generally supposed to have non-zero determinant. By diagonalizing the above matrices, we obtain the following fermion mass hierarchies;

$$m_u : m_c : m_t \simeq \epsilon^4 : \epsilon^2 : 1, \tag{6}$$

$$m_d : m_s : m_b \simeq \epsilon^2 : \epsilon : 1, \tag{7}$$

$$m_e : m_\mu : m_\tau \simeq \epsilon^2 : \epsilon : 1. \tag{8}$$

Now the compactification scale is supposed to be around the GUT scale and then the suppression factor becomes $\epsilon \sim 0.1$ when taking M_* as the typical string scale $\sim 10^{18}$ GeV (see Eq. (1)). This value is roughly consistent with the experimental mass eigenvalues of quarks and leptons in the large $\tan\beta$ case. Note that the larger hierarchy in the up-quark sector than in the down-quark and charged-lepton sectors is remarkably accomplished. Exactly speaking, the first generation down-quark and charged-lepton masses do not agree well with the experimental values. This point will be improved later, in that case more matter fields live in higher dimensions beyond the four.

Another important result of the above matrices is the mixing angles of quarks and leptons. It can be seen from the Yukawa matrices (2) and (3) that the quark mixing angles are small. This is due to the fact that both the up- and down-type left-handed quarks belong to the same **10** multiplets which have the hierarchical property in the present model. On the other hand, the left-handed charged leptons are in the $\mathbf{5}_i^*$ and there is no special structure among them. This fact indicates that in contrast to the quark sector, the lepton mixing angles are large which is actually required to solve the observed anomalous results in the neutrino experiments. As for the neutrino side, we do not have any conditions on the right-handed neutrinos fields, that is, we deal with them equally. In this situation, it is natural to naively assume an approximate permutation symmetry and then no hierarchical structure in the right-handed Majorana mass matrix. Then, the structure of the left-handed Majorana mass matrix can be determined by the properties of $\mathbf{5}_i^*$ only as long as the seesaw mechanism [15] is applied in order to have small neutrino masses. Consequently we expect the large mixing between generations in the neutrino side.

After all, we can have the large mixing angles in the lepton sector unless the accidental cancellation between the charged-lepton and neutrino mixing matrices occurs.

Here we comment on another possibility of having a hierarchical factor from the existence of extra dimensions, that is, the power-law effects from the renormalization group equations. For example, we consider a slightly different $SU(5)$ model than before. In this model, all three generation matters are now confined on the wall and instead of it, there is a new gauge singlet field θ which lives in the bulk. In addition, we assume a $U(1)$ flavor symmetry under which the θ field has charge -1 and the $\mathbf{10}_1$ and $\mathbf{10}_2$ also have charge $+2$ and $+1$, respectively, and all other fields have zero charge. In this model, hierarchical parameters are provided by the power-law evolutions of Yukawa couplings due to the presence of the KK excitations associated with the θ fields [3]. For example, the above $U(1)$ symmetry permits a superpotential term $y\,\theta^4\mathbf{10}_1\mathbf{10}_1H$ where y denotes a dimensionful 'Yukawa' coupling. This term induces the couplings of KK modes of θ to the matter field $\mathbf{10}_1$ on the wall. With these couplings, summing up the one-loop graphs which include the KK modes of θ induces a correction which is proportional to $y^2\,(M_*R)^{4n}$ to the anomalous dimension of $\mathbf{10}_1$. Here n denotes the number of extra dimensions that the θ feels, and the contribution of Higgs KK modes is neglected because it is a common contribution to all the anomalous dimensions of matter fields.[†] By rescaling y to the dimensionless Yukawa coupling $Y_{u_{11}}$, it can be easily seen that the correction to $Y_{u_{11}}$ becomes of order $(M_*R)^{-(2n+4)}$. This is just the low-energy value of $Y_{u_{11}}$ itself, assuming large Yukawa couplings at high-energy scale (the quasi fixed-point behavior [16]). By similar analyses of the other Yukawa couplings we obtain the same forms of matrices as (2)–(5), now by replacing the suppression factor $\epsilon \simeq (M_*R)^{-1/2}$ with $\epsilon' \equiv \epsilon^{(n+2)/2}$. In case $n=1$, for example, the requirement that $\epsilon' \sim 0.1$ implies $M_*R \sim 30$. Note that the hierarchical values of Yukawa couplings are realized as the quasi fixed-point values. This fact indicates that each Yukawa coupling is very large at high energy and in addition its low-energy value is determined independently of the high-energy informations. With these reasons a volume factor associated with the singlet θ is not the dominant feature in this approach. In this way, we can reproduce the same results as that in the previous scenario. This translation is actually possible for any matrix form. In addition, in the present model there is no standard gauge non-singlet fields in the bulk. This may be preferable from some phenomenological points of view (proton decay etc.).

The mass matrix forms of the Eqs. (2)–(5) have been already discussed before in the

[†]The Higgs anomalous dimensions can also be neglected if we assume that the Higgs fields feel some numbers of extra dimensions.

context of the $SO(10)$ grand unified theory [17] and the supersymmetric composite model [18]. In these models, the hierarchical parameters can be obtained by setting the other parameters or the compositeness scales of technicolor gauge groups. This fact usually requires some specific situations (the special couplings or gauge groups, etc.) in order to have hierarchical structure. The present mechanism of large volume factors from the extra dimensions, however, can always be applied in any (GUT) models, and has wide possibilities of solving other phenomenological problems.

In this way, we can have the roughly consistent predictions to the low-energy values of masses and mixings. In the present models, it is essential to accomplish the natural hierarchy that the **10** representation matters live in the bulk or have non-zero charges under some flavor symmetry. However, this situation has nothing to do with the neutrino mass matrix. The realistic mass difference for solving the solar neutrino problem, by the matter enhanced (MSW) oscillation solution [19] or the vacuum oscillation scenario [20], can be obtained by an accidental cancellation among the couplings or by introducing the other small parameters. In the following we discuss this point while considering the several possible improvements of the previous simple scenario.

First, we consider the case in which some more matter fields also feel the extra dimensions. In particular, we assume that the first generation 5-plet, $\mathbf{5}^*_1$, feels one (the fifth) extra dimension besides the $\mathbf{10}_{1,2}$. This induces further suppression factors and then we have the following Yukawa matrices;

$$Y_d \simeq \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \end{pmatrix}, \tag{9}$$

$$Y_e \simeq \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}. \tag{10}$$

The up-quark and neutrino Yukawa couplings are not affected by this change. On the other hand, the left-handed neutrino Majorana mass matrix takes the following form, assuming no constraint (no hierarchy) among the right-handed neutrino masses as mentioned before,

$$m_\nu^L \simeq \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \frac{v^2}{M_N}, \tag{11}$$

where M_N is a typical right-handed neutrino Majorana mass and v is the vacuum expectation value of the electroweak Higgs doublet. The mass eigenvalues of quarks and leptons

now become,

$$
\begin{aligned}
m_u : m_c : m_t &\simeq \epsilon^4 : \epsilon^2 : 1, && (12)\\
m_d : m_s : m_b &\simeq \epsilon^3 : \epsilon : 1, && (13)\\
m_e : m_\mu : m_\tau &\simeq \epsilon^3 : \epsilon : 1. && (14)
\end{aligned}
$$

This hierarchical structure is more realistic than before, namely, the down-quark and charged-lepton masses of the first generation are improved. Exactly speaking, however, the $O(1)$ coefficients (the Clebsch-Gordon coefficients) may be needed to reproduce the actual low-energy values in the down-quark and charged-lepton sector. Note that these mass matrix structures can also be obtained in the model with the power-law running effects as described before, in which the $\mathbf{5}^*_1$ field is now assumed to have charge $+1$ under the $U(1)$ flavor symmetry.

As for the neutrino masses and mixing, the mixing between the first and second generations is small $\sim O(\epsilon)$ and the mixing between the second and third generations is expected to be large $\sim O(1)$ in the neutrino and charged-lepton sectors. At first sight, the latter result seems to require rather fine-tunings among the Yukawa couplings. However, the matrix $Y_e Y_e^\dagger$, which the left-handed mixing matrix for the charged lepton can diagonalize, has the same form as m_ν^L (11). Since these two matrices are given by the Yukawa couplings squared, we can compare the degrees of tuning in diagonalizing these matrices. In the matrix form of Y_e (10) which is often discussed in the literatures it is usually assumed that the muon mass eigenvalue is reproduced by the tuning of couplings. The necessary order of fine-tuning is $(m_\mu/m_\tau)^2 \sim 10^{-3}$ from the experimental values. On the other hand, in the m_ν^L, we need a tuning of the order of $(\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm})^{1/2} \sim 10^{-1}$ to have the experimentally required mass-squared differences for the neutrino anomalies. Therefore, we can more naturally realize a mass hierarchy in the neutrino sector than that in the charged-lepton sector which is often assumed. In this situation, the masses of the light neutrinos from the above matrix (11) become,

$$
m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \simeq \epsilon^2 : (\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm})^{1/2} : 1 \simeq \epsilon^2 : \epsilon : 1. \qquad (15)
$$

For $\epsilon = (M_* R)^{-1/2} \sim 0.1$, this hierarchy shows the required mass eigenvalues for $M_N \sim 10^{14-15}$ GeV. This value of M_N is relatively larger than the typical intermediate scale favored by cosmology and astrophysics. But, there is a simple solution. That is, if the $\mathbf{5}^*$ fields feel more extra dimensions, M_N can become lower. More interestingly, this choice also reduces the down-quark and charge-lepton mass eigenvalues and could realize the

small $\tan\beta$ case. After all, we can see that the solar neutrino problem is solved by the small angle MSW solution between the first and second generations and the atmospheric neutrino anomaly, by the large mixing between the second and third generations. Other oscillation solutions are also possible. As mentioned in Ref. [18], if we assume that the 2×2 sub-matrix of m_ν^L has rank 1, it leads a bi-maximal solution in which the solar neutrino problem is solved by the large angle MSW solution instead of before. The vacuum oscillation scenario is not readily achieved because the large hierarchy between the second and third generations in the neutrino sector leads too small down-quark and electron masses. This may be improved if there are other origins of hierarchy in the right-handed Majorana mass matrix independently of the properties of right-handed neutrinos which, for example, comes from other singlet fields.

We finally discuss the possible existence of larger size of extra dimensions, $\sim O(\text{mm})$ and/or $\sim O(\text{TeV}^{-1})$ scale. In the models discussed so far, we have derived the light neutrino mass matrix by taking an assumption of the heavy Majorana masses and utilizing the seesaw mechanism. However, since the right-handed neutrinos are the standard gauge singlets they can propagate in these large extra dimensions without any problems. This can produce different and interesting phenomenology to neutrino physics. Consider an additional assumption than the previous $SU(5)$ models that the right-handed neutrinos $\mathbf{1}_i$ are also bulk fields. In this case, a large volume factor with these right-handed bulk neutrinos leads a large suppression of the neutrino Dirac masses and one may no longer need heavy fields to have small masses. Then it has been shown that the $O(\text{mm})$ size of extra dimensions are just the required order of magnitude for explaining the neutrino problems [10]. The detailed analyses in this framework show that the solar neutrino problem can be solved by the small angle MSW solution (the oscillation to the bulk neutrinos) but the large mixing angle for the atmospheric neutrino anomaly is excluded by the astrophysical limits [21]. However, in the present $SU(5)$ model, since the large mixing angle between the second and third generations can be obtained from the charged-lepton side we can expect to have a natural solution for both neutrino problems without heavy fields.

In this scenario, the gravity is also extended to higher dimensions. This implies that since the realm of quantum gravity is now open at rather lower scale M_*, the simple unification description at high-energy scale is no longer relevant. Then we suppose that the compactification scale R^{-1} of the extra dimensions which the standard non-singlet fields can feel is as low as order TeV, namely, a low-scale unification scenario [3]. We also assume

that the number of the large extra dimensions ($r \sim O(\mathrm{mm})$) is two[‡] in order to avoid the gravitational experimental limits and to obtain interesting neutrino phenomenology as mentioned above. Then it can be easily seen from the relation $M_{\mathrm{pl}}^2 = M_*^{\delta+2} R^{\delta-2} r^2$ that the remaining δ extra dimensions are consistently around the scale of M_*, which can be as low as TeV. We can use these freedom (the δ extra TeV^{-1}-sized dimensions) to have fermion mass hierarchy among the generations as described before. After all, we consider a model in which the matters corresponding to $\mathbf{10}_{1,2}$, etc. live in some of these R-sized extra dimensions as previously and the right-handed neutrinos and gravity live in larger r-sized dimensions. Since in this model the whole $SU(5)$ multiplets always together feel extra dimensions, the gauge coupling unification surely occurs in a similar way to Ref. [3]. As for the Yukawa couplings, the hierarchical structure is obtained in the same way as described before. The suppression volume factor is now numerically evaluated from the gauge coupling unification scale $\Lambda\,(\simeq M_*)$ as

$$\epsilon \simeq \frac{1}{(\Lambda R)^{n'/2}} \sim \frac{1}{\sqrt{20}}\,. \tag{16}$$

Note that in the above formula, n' denotes the maximal number of extra dimensions in which the standard gauge non-singlet fields live. This implies that ϵ in Eq. (16) is the minimum suppression factor accompanied with such bulk fields and the others are larger than ϵ. In addition, one more serious problem arises in this low-scale unification scenario; the proton-decay amplitude must be suppressed. In case no matter multiplet can propagate through more than four dimensions, several mechanisms to avoid the proton decay have been proposed; the (discrete) symmetries [3, 22, 23], some specific dynamics [9], and so on. Now, if only the $\mathbf{10}$ fields live in the bulk, some discrete symmetry could forbid the dimension five baryon number violating operators. However, naively the dimension more than six operators from the Kähler potentials are not forbidden and we would have to rely on other model-dependent mechanisms such as within the context of string theory. But there is a natural solution to these two problems. As discussed before we can construct the same Dirac mass matrices in the models with the RGE suppression factors (except for that of the neutrino which is suppressed by the very large volume factor). In this alternative construction, no gauge non-singlet matter lives in the bulk and clearly the proton can be stable to all-orders in perturbation theory if, for example, we impose the Z_2 parity symmetry under which the dangerous baryon number violating fields have odd parity [3]. Moreover, in the expression of the suppression factor (16), the number n' is now replaced

[‡]This is also needed to have three chiral families in a construction from the Type I string theory [22].

with that determined by the dimension of each higher dimensional operator including the bulk gauge-singlet field θ. Compared to the approach utilizing the volume factor ϵ, the 'unit' suppression factor becomes $\epsilon'(\equiv \epsilon^{(n+2)/2})$. E.g., for $n = 1$, $\epsilon' \simeq (1/\sqrt{20})^{3/2} \simeq 0.1$, which is just the required value for the observed hierarchies of fermion masses. In this way all the problems stated above can be cured in this alternative approach with the RGE effects.

In summary, we have demonstrated that in models with the (large) extra space-time dimensions, the realistic fermion mass hierarchy can be naturally obtained by the suppression factors originated from the existence of extra dimensions. In the models presented in this letter, not only the hierarchical structure but also the large mixing angle in the lepton sector can be realized. Of course, there are still very wide possibilities with these extra dimensions, and it is interesting to consider that the structure of the extra dimensions and fermion mass hierarchy may be strongly related.

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